277 (Mechanics) [1913, 196; 1919, 213]. Proposed by W. J. GREENSTREET, Burghfield Common, Berks., England.

Around a smooth fixed circular pulley is wound a massless inextensible string, and straight portions go to two free ends A and B to which masses are fastened. The mass at A is initially projected perpendicular to the string while the other is initially at rest. The length of the straight portion to the first mass is initially l and subsequently is r. Find the velocity of the second mass at that moment.

SOLUTION BY J. B. REYNOLDS, Lehigh University.

Let the masses initially at A and B be n and n', respectively. Let z be the distance n' has moved and r the distance from the pulley to n at any subsequent time; θ, the angle that r makes with its initial position l. If T is the tension in the string, the equations of motion are

$$T = -n(\ddot{r} - r\dot{\theta}^2), \qquad \frac{d}{dt}(r^2\dot{\theta}) = 0, \qquad T = n'\ddot{z}.$$

The second of these gives, $r^2\dot{\theta} = \text{const} = v_0 l$, v_0 being the initial velocity of projection. Again, since $r = z + \text{constant}$, $\ddot{z} = \ddot{r}$ and eliminating T between the first and third equations, we have

$$(n + n')\ddot{r} = nr\dot{\theta}^2 = \frac{nv_0^2l^2}{r^3},$$

or, since

$$\ddot{r} = \dot{r}\frac{d\dot{r}}{dr},$$

we get upon integrating,

$$(n + n')\dot{r}^2 = -\frac{n\,v_0^2l^2}{r^2} + nv_0^2 = nv_0^2\left(1 - \frac{l^2}{r^2}\right).$$

Therefore, since the velocity $\dot{z}$ of $n' = \dot{r}$ at any time, we have

$$\dot{z} = \frac{v_0}{r}\sqrt{\frac{n}{n + n'}}\sqrt{r^2 - l^2}.$$

2700 [1918, 215; 1919, 215, 365]. Proposed by the late ARTEMAS MARTIN.

In a factory 250 men are paid an average wage of \$15 each per week. The men are paid unequally, the wages being \$20, \$16, \$10, and \$8 per week respectively, for different classes of work. How many are employed at each rate of pay?

NOTE.—I am told that this question was set in a Civil Service examination paper to be worked by arithmetic. 2,896 answers have been found. Are there any more?

III. REMARKS BY H. S. UHLER, Yale University.

On page 366 of the October issue the interesting suggestion is made by Professor D. N. Lehmer that: "The actual number might be obtained by counting the number of lattice points in this quadrilateral." I have followed this suggestion by counting along lines parallel to the axis of ordinates. Let the sides of the trapezium the equations of which are $3X + 2Y - 625 = 0$, $X - Y = 0$, $875 - 4X - 2Y = 0$, and $Y = 0$ be denoted respectively by I, II, III, and IV. The coördinates of the vertices are (125, 125), ($145\frac{5}{6}$, $145\frac{5}{6}$), ($208\frac{1}{3}$, 0), and ($218\frac{3}{4}$, 0). The number of lattice points limited by sides I and II is $1 + 3 + 6 + 8 + 11 + 13 + 16 + 18 + 21 + 23 + 26 + 28 + 31 + 33 + 36 + 38 + 41 + 43 + 46 + 48 + 51 = 541$. The number limited by sides I and III is $2(52 + 51 + \cdots + 23 + 22) + 21 = 2315$. The number limited by sides IV and III is $20 + 18 + \cdots + 4 + 2 = 110$. The total number of lattice points within and on the periphery of the trapezium is 2966, which is identically the value already given [*1919*, 216]. The numbers of points on sides I, II, III, and IV are 42, 21, 0, and 10, respectively. The sum of the distinct points, 72, verifies my earlier remark [*1919*, 216] that: "The number of solutions involving one or more zeros is 72." I should like to know whether there is any general significance to the fact that the "error" 36 [$2966 - 2929\frac{11}{16}$] is exactly one-half of 72, *i.e.*, the total number of peripheral solutions.